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ABSTRACT

An understanding-based approach to teaching arithmetic word problems is used in the Training Arithmetic Problem Solving Skills (TAPS) research project, for which four semantic schemas or problem representations have been revised and adopted: Combine, Compare, Change, and Vary. It is hypothesized that a good problem solver identifies the schema of the problem, instantiates the slots with problem information, verifies the schema identification, executes the appropriate procedural attachment to generate the numerical answer, and checks to see whether the solution is appropriate. In the case of a complex problem, the problem solver also has to recognize the goal of the problem and use memory-management skills to store and retrieve information as required in the solution process. Researchers plan to teach these skills explicitly in the TAPS instructional program, which is designed for use by elementary students who have reading and computational competencies, but perform poorly in solving word problems. An individualized curriculum will be developed for each student on the basis of a diagnostic test which will identify deficient skill areas, and the new skill will be introduced via direct didactic instruction and practiced without speed pressure. Performance will be further improved through intensive drill and practice monitored by an intelligent tutoring system. Seven references are listed. (MES)

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Application of Schema Theory to the Instruction of
Arithmetic Word Problem Solving Skills

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Running head: ARITHMETIC WORD PROBLEM SOLVING SKILLS

Abstract

Word problems is an important area in arithmetic education. Conventional approaches for teaching word problems have serious drawbacks and may develop unreliable skills. The TAPS project instead uses an understanding-based approach. Four schemas of word problems are identified and analyzed. An information-processing model of word problem solving is also hypothesized. A diagnostic test and instructional program are being developed based on the schema theory. A powerful intelligent tutoring system for drill-and-practice is also incorporated in the instructional program.

Introduction

Word-problem solving is recognized as an important and special area in arithmetic education. As the name indicates, it requires problem solving skills. Students have to integrate much knowledge and rule-using to perform this skill. More significance of word problems comes from another feature of them. Word problems are verbal simulation of arithmetic applications to real world situations. By doing word problems, students prepare for using their arithmetic skills in the real world.

Conventional Instructional Approach

The common practice of word-problem instruction is operation- and number-oriented. The instruction is given in the context of arithmetic operations for different types of numbers. After students have learned to perform an operation with a certain type of number, e.g., natural numbers, they practice solving word problems which contain the type of number and require the operation. Beside practicing applying the arithmetic operations they have learned, students are usually taught to use certain key words to identify the correct operations for word problems.

The conventional approach to teaching word problems has some serious drawbacks. The required arithmetic operation required by a word problem is dependent on which information the problem requests. The solution procedure may also be dependent on this factor. Therefore knowledge based on arithmetic operations or number types is not a reliable basis of word-problem solving skills. The key-word method has the same weakness. Key words are sometimes valid cues of solutions and sometimes not. Students can be "tricked" to choose an incorrect operation if they rely totally on key words to solve word problems (Webb, 1984). The key-word method is also ineffective for complex problems.

Understanding-Based Approach

Recently researchers emphasize the importance of understanding in word problem solving skills (Mayer, 1983; Kintch & Greeno, 1985; Derry, Hawkes, & Tsai, in press).

Understanding is hypothesized to be the construction of an appropriate problem representation upon which problem solving processes can operate successfully (Greeno, 1977).

Semantic schemas as problem representations for simple word problems have been described by researchers (Riley, Greeno, & Heller, 1983; Marshall, 1985). Four schemas proposed were revised and adopted for the TAPS (Training Arithmetic Problem Solving Skills) research project. These schemas are Combine, Compare, Change, and Vary. A schema is depicted as a network of linked sets. The basic building blocks of a schema are sets (Kintch and Greeno, 1985). A set is a collection of objects. We can analyze a set into information of kind, specification, unit, and quantity of the objects in the set. Information on procedural operations are attached to a schema to indicate the appropriate operations for solving unknown sets of the schema.

 Insert Figure 1 about here

Figure 1 shows the diagram of the Change schema. The Change schema represents a situation in which some objects are transferred from one set to the other. The Start sets and the Result sets contain information of the sets before and after the transfer action, respectively. And the transfer set contains information of the objects transferred.

Here is an example of a word problem depicting the Change schema: "Joe had 15 marbles. He gave Tom 3 marbles. How many marbles does Joe have now?" In this problem 15 marbles are the Start set. Its specification is belonging to Joe at an earlier time, object is certainly marble, unit is none, and quantity is 15. The Transfer set is the three marbles. The Result set is the marbles that Joe has now. The quantity of the Result set is the requested goal of this problem. Because Joe is the giver and the Result set is the goal, the required operation is subtraction.

It is hypothesized that in solving a word problem, a good problem solver identifies the schema of the problem, instantiates the slots with problem information and verifies the

schema identification, executes the appropriate procedural attachment to generate the numerical answer, and checks to see whether the solution is appropriate. In the case of a complex problem, the problem solver also has to recognize the goal of the problem, chain the schemas in the problem, and use memory-management skills to store and retrieve information as required in the solution process. As a contrast, a poor problem solver suffers from some of the following skill deficiencies: They do not reliably identify schemas of problems; They do not instantiate problem schemas successfully; They do not reliably identify the correct operation based on position of the unknown set in the schema identified; And they are distracted easily by goal-irrelevant information in a problem. A poor problem solver may also suffer from schema-chaining and/or memory management deficiencies when they work on complex problems.

The TAPS Instructional Approach

In light of the above theoretical analysis of problem-solving process, the authors and colleagues plan to teach the component skills explicitly in the TAPS instructional program. Figure 2 shows the map of the instructional program which represents component skills of an "expert" problem solver. The lower-level objectives represent training of basic schema knowledge and skills. The higher-level objectives are chaining, strategic, and motivational skills which are necessary for the terminal objective of solving multi-schema problems.

 Insert Figure 2 about here

The target population of the TAPS program are elementary school students. These students have reading and computational competencies, but perform poorly in solving word problems. Students entering TAPS will be given a diagnostic test to determine their competencies on objectives of the instructional map. A student who can not perform a skill will be categorized as "deficient" with respect to that skill. When performance is accurate but slow, the student is characterized as an "informed novice." If a student can

perform the skill under speed stress reliably, s/he will be labeled as "expert" for that skill.

An individualized curriculum will then be devised by selecting skills from the instructional map that must be learned from scratch or practiced to proficiency. For areas in which the student is deficient, each new skill will be introduced via direct, didactic instruction. Once the skill is learned in a declarative sense, it must then be practiced without speed pressure until performance reaches the level of "informed novice."

Novice performance of a student will be improved further through intensive drill-and-practice monitored by an intelligent tutoring system (ITS) until the rapid, automated performance style of an "expert" has evolved. In collaboration with the authors, the ITS is being developed by Dr. Lois Hawkes and her students in Florida State University. The ITS will be a complex system integrating many component modules. The system will be able to monitor a student and diagnose causes of his or her difficulties, generate problems and prompts to suit the student's needs, solve problems generated and construct a comprehensive solution tree to monitor the student's solution path, and communicate with the student via a natural language interface. Whereas the primary role of the ITS in TAPS is to deliver intensive drill-and-practice, the ITS could eventually control diagnostic testing and the didactic instruction as well.

Conclusion

TAPS represents a new instructional approach explicitly based on schema and information processing theory. New and powerful technology is also being incorporated to implement the instructional program. Experiences from the TAPS project will not only help to advance instructional science but also improve understanding in the area of human cognition.

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Author Notes

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Figure Caption

Figure 1. The Change schema.

Figure 2. TAPS instructional map.

- slot to be verified
 slot to be instantiated
 slot to be instantiated (not necessarily)
 slot to be instantiated (either or both)
 anchoring point for procedural attachments

Spec A: Specification of set A

Q(A): Quantity of set A



